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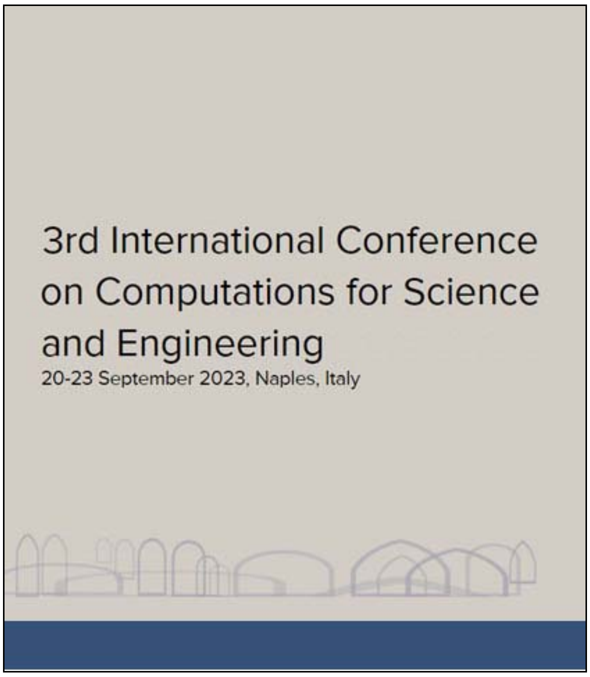

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The Effect of Depth's Inclusions on Vibration Attenuation while Using Seismic Metamaterial

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ABSTRACT

Seismic metamaterials are used in different fields of civil engineering to contrast the propagation of the vibration to protect urban areas from their effect. Generally, the type of metamaterials considered in literature have fully embedded inclusion in the soil, or are pillars (inclusion) placed at the surface. This study aims to investigate the effects that the depth of the inclusion has on the band gaps that can be obtained as well as the level of transmission that can be achieved. This is done by parametric investigation of the different depths by the implementation, using the finite element software COMSOL Multiphysics, of two models one for the unit-cell and the one for the array of the seismic metamaterial.

Keywords: *Vibration Waves, Unit-cell, Seismic Metamaterial, Mitigation measures*

1. INTRODUCTION

Urban areas in modern times face an increasing challenge - induced vibrations stemming from sources like road and rail traffic, earthworks, and natural hazards, such as earthquakes [1-3]. These vibrations pose risks to both structures and residents. To mitigate these effects and allow residents to carry out daily activities without disturbance or health risks, researchers have turned to metamaterials, a novel concept receiving considerable attention in vibration attenuation. Metamaterials are engineered materials with unique properties not found in conventional materials and find applications in acoustics and optics. In civil engineering, they are harnessed to counter mechanical vibrations from sources like traffic, machinery, and earthquakes [4-6].

This paper presents a comparative study of two types of mechanical metamaterials: seismic metamaterials, which are fully embedded structures, depicted in Figure 1(c), and resonant metamaterials (also known as natural metamaterials), reported in Figure 1(d), which involve naturally occurring materials, often using trees as resonators at the surface. The research evaluates the impact of various geometrical properties on the thickness, and frequency range of the first band-gap. Furthermore, it explores the transition from fully embedded inclusions to resonator pillars at the surface, introducing the concept of semi-embedded metamaterials, as illustrated in Figure 1(e). The study concludes with practical insights for selecting the most suitable mechanical metamaterial configuration based on the desired frequency range for vibration attenuation.

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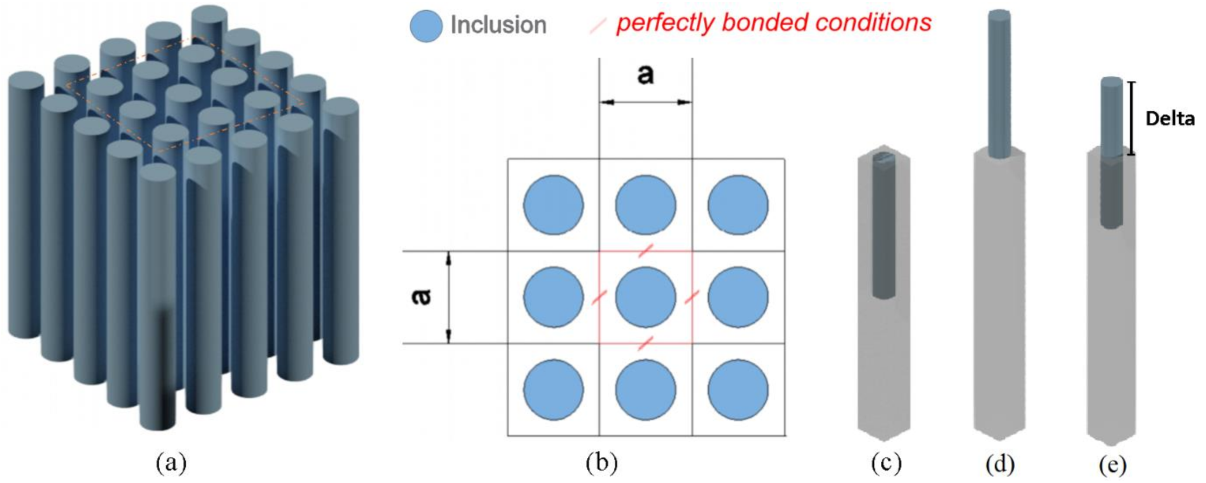

Figure 1. Graphical representation of the mechanical metamaterial (a) mechanical metamaterial 3D view, (b) mechanical metamaterial plan, (c) 3D view of the seismic metamaterial unit-cell, (d) 3D view of the resonant metamaterial unit-cell, and (e) 3D view of the transitional metamaterial unit-cell.

2. MECHANICAL METAMATERIALS

Metamaterials are materials or structures composed of periodic unit blocks, have gained prominence in wave attenuation research. These materials enable the control of various waves, transcending differences in their nature, such as longitudinal or transverse, elastic or electric [7]. Researchers have extended the application of metamaterials to address mechanical vibrations induced by ground transportation, construction machinery, and low-amplitude earthquakes, collectively referred to as seismic waves. These waves propagate through the soil on a much larger scale.

Mechanical metamaterials, often called periodic barriers, are elastic structures that incorporate scattering inclusions within a material [8]. These structures exhibit periodicity, with various forms including one-dimensional, two-dimensional, and three-dimensional configurations. Periodic barriers can enhance the absorption of elastic waves, improving applications like seismic wave attenuation and wave-absorbing interfaces by isolating vibrations within specific frequency ranges called band-gaps.

This study primarily focuses on two-dimensional mechanical metamaterials used in civil engineering applications, categorizing them as seismic metamaterials when inclusions are embedded in the soil [9], and resonant (natural) metamaterials when inclusions act as resonators above the soil [5]. These metamaterials aim to mitigate vibration responses in various engineering contexts. The study emphasizes a geometrical parametric investigation of these mechanical metamaterials, offering insights into their attenuation capabilities, with consideration for the use of cylindrical inclusions and diverse material properties.

2.1 Wave propagation in periodic barriers

The periodic barrier is the result of the repetition of a pattern of an elementary cell (see Figure 1(a) and (b)), in one or more directions. The periodicity conditions imposed at the border of this pattern make it possible to limit its very small-scale analysis using the Floquet-Bloch theory [10]. An example of a periodic medium in two dimensions which illustrates the Floquet-Bloch transformation is graphically depicted in Figure 1. Therefore, the Floquet-Bloch transformation is a tool for reducing the size of analytical (and therefore numerical) models of a periodic medium by limiting the study to the analysis of the problem at the eigenvalues of the elementary cell alone. By joining the wave propagation theory for isotropic media and the Floquet-Bloch transformation in the case of a square unit cell (i.e. $a = ax = ay$) with a period "a", and applying the periodic boundary conditions, the dispersion relations can be obtained as:

$$\boldsymbol{u}(r + a) = \boldsymbol{u}(r)e^{ika}$$

where a is the lattice vector, and $k = [k_x, k_y]$ the wave vector. From the combination of the latter equation and the wave propagation equation for isotropic material is possible to obtain the following dispersion equation in terms of eigenvalue:

$$(\boldsymbol{K} - \omega^2 \boldsymbol{M})\boldsymbol{u}$$

Where **K** and **M** are the stiffness matrix and the mass matrix, respectively. The dispersion relation reported in the dispersion equation is a function of the wave vector **k** and the eigenfrequency ω. Therefore, to find the frequency modes, the wave vector should sweep along Γ, X, M, Γ (see Figure 2b) to see its change across the boundary of the first irreducible Brillouin zone. The band-gaps are represented by all the wave vectors for which the corresponding frequencies do not exist.

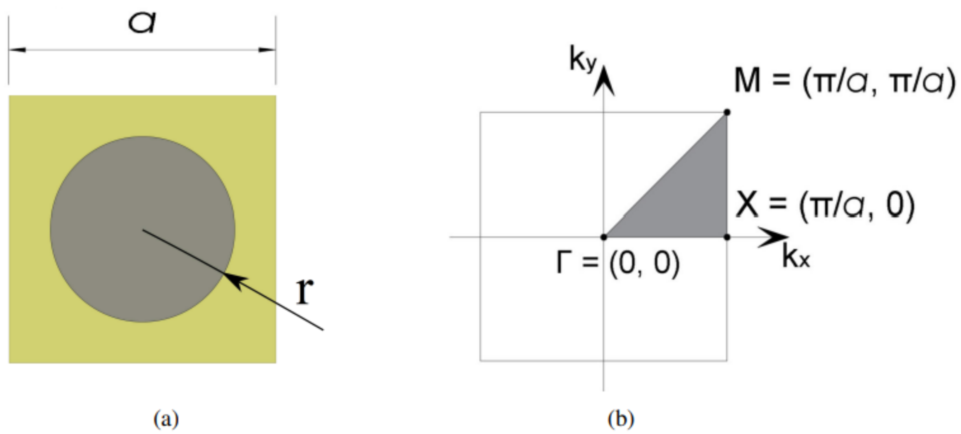

Figure 2. A primitive cell of the periodic medium in physical space: (a) unit-cell corresponding to an infinite medium along two directions and (b) reciprocal unit-cell on which the contour of the Brillouin irreducible zone is represented in the light grey triangle of vertices Γ, X, M.

3. NUMERICAL MODELS

In this study, the investigation of wave propagation in a two-dimensional periodic structure, consisting of cylindrical inclusions on a semi-infinite substrate, is conducted through a numerical model based on finite element methods. The unit cell is constructed using COMSOL Multiphysics software, which defines the wavevector within the Brillouin zone by solving the wave dispersion equation. This approach allows for the generation of dispersion curves and the identification of relative band-gaps. The unit cell configuration involves a square lattice with cylindrical inclusions, characterized by parameters such as period (α), radius (r), inclusion height (h), and the depth of inclusion within the hosting material (δ). The model assumes periodicity in both x and y directions, employing Perfectly Matched Layers (PMLs) to prevent wave reflections. Validation of the developed model is achieved by comparing it to previous numerical models in the literature, with validation studies for seismic metamaterials and resonant metamaterials against works by Achaoui et al. [11] and Khelif et al. [12], respectively.

3.1 Reference numerical models

In the study of metamaterial attenuation regarding Rayleigh wave propagation, some assumptions have been made. These assumptions include utilizing a unit-cell consisting of a square substrate with a cylindrical pillar as the inclusion, implementing a 3D hexahedral mesh for the inclusion and a tetrahedral mesh for the substrate, and employing Perfectly Matched Layers (PMLs) to prevent reflections at domain boundaries and replicate the semi-infinite behaviour of the substrate. To facilitate the parametric investigation discussed in Section 5 and to clarify the contribution of each parameter, a reference unit-cell is used. The parameters for this reference unit-cell, with a period (a) of 2 [m], are detailed in Table 1, as per the research conducted by Ouakka et al. for steel inclusions [6]. Based on the material and geometric characteristics outlined in Table 1, the initial fifteen Bloch modes are calculated for the wave vector k along the edges of the Brillouin zone (Γ, X, M, Γ). Notably, the surface modes of interest are found within the sound cone region, while the bulk wave modes are situated outside of this zone.

4. RESULTS AND DISCUSSION

This section presents briefly the outcomes of our parametric investigation, aiming to gauge the influence of various parameters on the attenuation of the basic unit-cell. We explore different inclusion geometries and evaluate their impact on the first bandgap, employing the numerical model outlined in Section 3. Our investigation covers two parameters: the length of the inclusion above the soil, ranging from 0 (seismic metamaterial) to 10 [m] (resonant metamaterial) with 2 [m] increments (yielding six configurations), and the inclusion's radius, ranging from 0.2 [m] to 0.8 [m] in 0.2 [m] increments (resulting in four configurations). The obtained first band-gabs for all the configurations are reported in Figure 3.

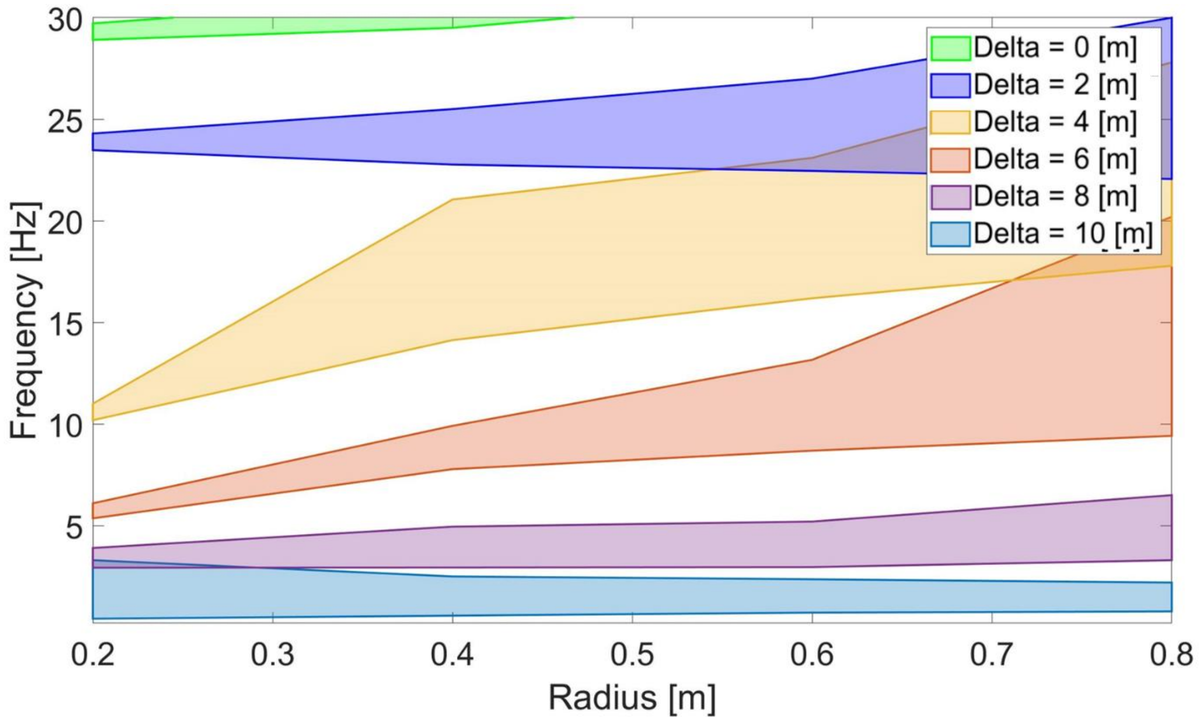

Figure 3. First band-gap evolution with respect to the radius and depth of the embedded part.

The analysis reveals that the transition from resonant metamaterial to seismic metamaterial corresponds to a first band-gap located at higher frequencies. At the same time varying the inclusion radius demonstrates that larger radii result in wider dispersion curve spacing within surface modes.

Additionally, for seismic metamaterial configurations, larger radii enhance wave attenuation due to increased inclusion material stiffness.

5. CONCLUSIONS

In urban areas, addressing induced vibrations remains a primary challenge for engineers, as creating a comfortable living environment for residents inside buildings is crucial. The adoption of novel mitigation techniques, particularly those based on metamaterials, is gaining prominence as an effective solution for mitigating these vibrations. This study delves into the fundamental principles of metamaterials and explores the two primary configurations in civil engineering: seismic metamaterials and resonant (natural) metamaterials. It investigates how these configurations can counteract wave propagation resulting from various sources of induced vibrations in urban settings. The investigation includes a parametric analysis of the unit cell's geometric properties.

The results from the parametric investigation reveal a close connection between the geometric parameters of the inclusions and the attenuation levels specific to each metamaterial configuration. Notably, the transition from resonant metamaterial to seismic metamaterial, achieved by varying the depth of the embedded part, leads to higher frequency values for the band-gap as the embedded part's length grows. Furthermore, varying the radius generates wider band-gaps as the radius increases. This research represents a novel approach that warrants further exploration to comprehensively understand the intricate relationships between these parameters and their impact on attenuation levels.

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